National Centre for Excellence in the Teaching of Mathematics
MathsHUBS


# Teaching for Mastery

## Questions, tasks and activities to support assessment

## Year 6

Mike Askew, Sarah Bishop, Clare Christie, Sarah Eaton, Pete Griffin, Debbie Morgan and Robert Wilne

Oxford OWL
OXFORD UNIVERSITY PRESS

# Contents



## Acknowledgements:




Cover photograph by Suzy Prior

Photographs by shcreative p.4; Suzy Prior p. 5

The Publisher would like to thank the following for permission to reproduce photographs: ARK Atwood Primary Academy, St Boniface RC Primary School and Campsbourne Infant and Junior School

The authors would like to thank Jane Imrie, of the NCETM, for her advice and support in reviewing the materials.

# About the authors

**Mike Askew** is Professor of Mathematics Education, the University of the Witwatersrand, Johannesburg. Mike has directed many research projects, including the influential 'Effective Teachers of Numeracy in Primary Schools', and was deputy director of the five-year Leverhulme Numeracy Research Programme. Mike's research has been widely published both in the academic arena and as books and resources for teachers.

**Pete Griffin** works at a national level as Assistant Director for the National Centre for Excellence in the Teaching of Mathematics. Pete has experience as a secondary teacher, Advisory Teacher, and lecturer in Mathematics Education at the Open University. Pete has worked with QCA and the National Strategies and has written and developed a wide range of teacher professional development materials.

**Sarah Eaton** is an Assistant Headteacher and Year 6 teacher. Sarah has been a Mathematics SLE with the Affinity Teaching School Alliance for four years, enabling her to lead CPD across the alliance. Sarah has been part of a Mathematics research project in Shanghai and Finland, and has been part of the KS2 teacher panel for the 2016 Maths tests.

**Debbie Morgan** holds a national role as Director of Primary Mathematics at the National Centre for Excellence in the Teaching of Mathematics. Debbie has experience as a primary teacher, Headteacher, Mathematics Advisor, Senior Lecturer in Mathematics Education and Director of a Mathematics Specialist Teacher Programme. Debbie currently provides advice and expertise to the DfE to support the implementation of the Primary Mathematics Curriculum.

**Robert Wilne** is Director of Secondary Mathematics at the National Centre for Excellence in the Teaching of Mathematics. He read Maths at Cambridge, and has taught in independent and state schools. Robert was the founding Headteacher of the London Academy of Excellence, the first sixth form free school. He has also been a school governor, in Tottenham and Newham.

**Sarah Bishop** is an Assistant Headteacher and Year 2 teacher with experience as a Primary Strategy Maths Consultant. She is currently a Mathematics SLE with Affinity Teaching School Alliance and has delivered CPD and school-to-school support as part of this role. Sarah has been involved in making the NCETM videos to support the National Curriculum and is part of the DfE Expert Group for Mathematics. More recently, Sarah has taken on the role of Primary Lead for the East Midlands South Maths Hub.

**Clare Christie** is a primary teacher and Maths Leader. Clare is also a Mathematics SLE, supporting schools with Maths teaching and learning. Clare is primary lead of the Boolean Maths Hub and a member of the ACME Outer Circle.

# Introduction

In line with the curricula of many high performing jurisdictions, the National curriculum emphasises the importance of all pupils mastering the content taught each year and discourages the acceleration of pupils into content from subsequent years.

The current National curriculum document[1] says:

*'The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.'* (National curriculum page 3)

Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency, as described in the aims of the National curriculum:

*'The national curriculum for mathematics aims to ensure that all pupils:*

- *become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately*
- ***reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language*
- *can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.'* (National curriculum page 3)

**1.** Mathematics programmes of study: key stages 1 and 2, National curriculum in England, September 2013, p3

Assessment arrangements must complement the curriculum and so need to mirror these principles and offer a structure for assessing pupils' progress in developing mastery of the content laid out for each year. Schools, however, are only '*required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study*' (National curriculum page 4). Schools should identify when they will teach the programmes of study and set out their school curriculum for mathematics on a year-by-year basis. The materials in this document reflect the arrangement of content as laid out in the National curriculum document (September 2013).

These *Teaching for Mastery: Questions, tasks and activities to support assessment* outline the key mathematical skills and concepts within each yearly programme and give examples of questions, tasks and practical classroom activities which support teaching, learning and assessment. The activities offered are not intended to address each and every programme of study statement in the National curriculum. Rather, they aim to highlight the key themes and big ideas for each year.

## Ongoing assessment as an integral part of teaching

The questions, tasks, and activities that are offered in the materials are intended to be a useful vehicle for assessing whether pupils have mastered the mathematics taught.

However, the best forms of ongoing, formative assessment arise from well-structured classroom activities involving interaction and dialogue (between teacher and pupils, and between pupils themselves). The materials are not intended to be used as a set of written test questions which the pupils answer in silence. They are offered to indicate valuable learning activities to be used as an integral part of teaching, providing rich and meaningful assessment information concerning what pupils know, understand and can do.

The tasks and activities need not necessarily be offered to pupils in written form. They may be presented orally, using equipment and/or as part of a group activity. The encouragement of discussion, debate and the sharing of ideas and strategies will often add to both the quality of the assessment information gained and the richness of the teaching and learning situation.

## What do we mean by mastery?

The essential idea behind mastery is that ***all children***[2] need a ***deep*** understanding of the mathematics they are learning so that:

- future mathematical learning is built on solid foundations which do not need to be re-taught;
- there is no need for separate catch-up programmes due to some children falling behind;
- children who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

There are generally four ways in which the term mastery is being used in the current debate about raising standards in mathematics:

1. **A mastery approach**: a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time. Pupils are neither 'born with the maths gene' nor 'just no good at maths'. With good teaching, appropriate resources, effort and a 'can do' attitude all children can achieve in and enjoy mathematics.
2. **A mastery curriculum**: one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them. There is no such thing as 'special needs mathematics' or 'gifted and talented mathematics'. Mathematics is mathematics and the key ideas and building blocks are important for everyone.
3. **Teaching for mastery**: a set of pedagogic practices that keep the class working together on the same topic, whilst at the same time addressing the need for all pupils to master the curriculum and for some to gain greater depth of proficiency and understanding. Challenge is provided by going deeper rather than accelerating into new

---

2. Schools in England are required to adhere to the 0-25 years SEND Code of Practice 2015 when considering the provision for children with special educational needs and/or disability. Some of these pupils may have particular medical conditions that prevent them from reaching national expectations and will typically have a statement of Special Educational Needs/ Education Health Care Plan. Wherever possible children with special educational needs and/or a disability should work on the same curriculum content as their peers; however, it is recognised that a few children may need to work on earlier curriculum content than that designated for their age. The principle, however, of developing deep and sustainable learning of the content they are working on should be applied.

mathematical content. Teaching is focused, rigorous and thorough, to ensure that learning is sufficiently embedded and sustainable over time. Long term gaps in learning are prevented through speedy teacher intervention. More time is spent on teaching topics to allow for the development of depth and sufficient practice to embed learning. Carefully crafted lesson design provides a scaffolded, conceptual journey through the mathematics, engaging pupils in reasoning and the development of mathematical thinking.

4. **Achieving mastery of particular topics and areas of mathematics**. Mastery is not just being able to memorise key facts and procedures and answer test questions accurately and quickly. It involves knowing 'why' as well as knowing 'that' and knowing 'how'. It means being able to use one's knowledge appropriately, flexibly and creatively and to apply it in new and unfamiliar situations.[3] The materials provided seek to exemplify the types of skills, knowledge and understanding necessary for pupils to make good and sustainable progress in mastering the primary mathematics curriculum.

## Mastery and the learning journey

Mastery of mathematics is not a fixed state but a continuum. At each stage of learning, pupils should acquire and demonstrate sufficient grasp of the mathematics relevant to their year group, so that their learning is sustainable over time and can be built upon in subsequent years. This requires development of depth through looking at concepts in detail using a variety of representations and contexts and committing key facts, such as number bonds and times tables, to memory.

Mastery of facts, procedures and concepts needs time: time to explore the concept in detail and time to allow for sufficient practice to develop fluency.

Practice is most effective when it is intelligent practice,[4] i.e. where *the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity.* (Gu 2004[5]) The examples provided in the materials seek to exemplify this type of practice.

## Mastery and mastery with greater depth

Integral to mastery of the curriculum is the development of deep rather than superficial conceptual understanding. '*The research for the review of the National Curriculum showed that it should focus on "fewer things in greater depth", in secure learning which persists, rather than relentless, over-rapid progression.*'[6] It is inevitable that some pupils will grasp concepts more rapidly than others and will need to be stimulated and challenged to ensure continued progression. However, research indicates that these pupils benefit more from enrichment and deepening of content, rather than acceleration into new content. Acceleration is likely to promote superficial understanding, rather than the true depth and rigour of knowledge that is a foundation for higher mathematics.[7]

Within the materials the terms *mastery* and *mastery with greater depth* are used to acknowledge that all pupils require depth in their learning, but some pupils will go deeper still in their learning and understanding.

Mastery of the curriculum requires that all pupils:

- use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
- recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
- have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

---

3. Helen Drury asserts in 'Mastering Mathematics' (Oxford University Press, 2014, page 9) that: 'A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically with the concept so that they can independently apply it to a totally new problem in an unfamiliar situation.'

4. Intelligent practice is a term used to describe practice exercises that integrate the development of fluency with the deepening of conceptual understanding. Attention is drawn to the mathematical structures and relationships to assist in the deepening of conceptual understanding, whilst at the same time developing fluency through practice.
5. Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In Lianghuo, F., Ngai-Ying, W., Jinfa, C., & Shiqi, L. (Eds.) How Chinese learn mathematics: Perspectives from insiders. Singapore: World Scientific Publishing Co. Pte. Ltd. page 315.
6. Living in a Levels-Free World, Tim Oates, published by the Department for Education https://www.tes.co.uk/teaching-resource/living-in-a-levels-free-world-by-tim-oates-6445426
7. This argument was advanced by the Advisory Committee for Mathematics Education on page 1 of its report: Raising the bar: developing able young mathematicians, December 2012.

A useful checklist for what to look out for when assessing a pupil's understanding might be:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)[8]
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;
- make use of it in various ways, including in new situations.[9]

Developing mastery with greater depth is characterised by pupils' ability to:

- solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
- independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.

The materials seek to exemplify what these two categories of *mastery* and *mastery with greater depth* might look like in terms of the type of tasks and activities pupils are able to tackle successfully. It should, however, be noted that the two categories are not intended to exemplify differentiation of activities/tasks. Teaching for mastery requires that all pupils are taught together and all access the same content as exemplified in the first column of questions, tasks and activities. The questions, tasks and activities exemplified in the second column might be used as deepening tasks for pupils who grasp concepts rapidly, but can also be used with the whole class where appropriate, giving all children the opportunity to think and reason more deeply.

## National curriculum assessments

National assessment at the end of Key Stages 1 and 2 aims to assess pupils' mastery of both the content of the curriculum and the depth of their understanding and application of mathematics. This is exemplified through the content and cognitive domains of the test frameworks.[10] The content domain exemplifies the minimum content pupils are required to evidence in order to show mastery of the curriculum. The cognitive domain aims to measure the complexity of application and depth of pupils' understanding. The questions, tasks and activities provided in these materials seek to reflect this requirement to master content in terms of both skills and depth of understanding.

## Final remarks

These resources are intended to assist teachers in teaching and assessing for mastery of the curriculum. In particular they seek to exemplify what depth looks like in terms of the types of mathematical tasks pupils are able to successfully complete and how some pupils can achieve even greater depth. A key aim is to encourage teachers to keep the class working together, spend more time on teaching topics and provide opportunities for all pupils to develop the depth and rigour they need to make secure and sustained progress over time.

**8.** The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See https://www.ncetm.org.uk/resources/44565 (free registration required) for an introduction to this approach.

**9.** Adapted from a list in 'How Children Fail', John Holt, 1964.

**10.** 2016 Key stage 1 and 2 Mathematics test frameworks, Standards and Assessments Agency www.gov.uk/government/collections/national-curriculum-assessments-test-frameworks

# The structure of the materials

The materials consist of PDF documents for each year group from Y1 to Y6. Each document adopts the same framework as outlined below.

The examples provided in the materials are only indicative and are designed to provide an insight into:

- How mastery of the curriculum might be developed and assessed;
- How to teach the same curriculum content to the whole class, challenging the rapid graspers by supporting them to go deeper rather than accelerating some pupils into new content.

The assessment activities presented in both columns are suitable for use with the whole class. Pupils who successfully answer the questions in the left-hand column (Mastery) show evidence of sufficient depth of knowledge and understanding. This indicates that learning is likely to be sustainable over time. Pupils who are also successful with answering questions in the right-hand column (Mastery with Greater Depth) show evidence of greater depth of understanding and progress in learning.

This section lists a selection of key National Curriculum programme of study statements. The development and assessment of these is supported through the questions, tasks and activities set out in the two columns below.

This section lists a selection of key ideas relevant to the selected programme of study statements.

**Number and Place Value**

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across 0
- solve number and practical problems that involve all of the above

**The Big Ideas**

For whole numbers, the more digits a number has, the larger it must be: any 4-digit whole number is larger than any 3-digit whole number. But this is not true of decimal numbers: having more digits does not make a decimal number necessarily bigger. For example, 0·5 is larger than 0·35.

Ordering decimal numbers uses the same process as for whole numbers ie we look at the digits in matching places in the numbers, starting from the place with the highest value ie from the left. The number with the higher different digit is the higher number. For example, 256 is greater than 247 because 256 has 5 tens but 247 has only 4 tens. Similarly 1·0843 is smaller than 1·524 because 1·0843 has 0 tenths but 1·524 has 5 tenths.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

| Mastery | Mastery with Greater Depth |
|---|---|
| Think about the number 34 567 800.<br><br>Say this number aloud.<br>Round this number to the nearest million.<br><br>What does the digit '8' represent?<br>What does the digit '7' represent?<br><br>Divide this number by 100 and say your answer aloud.<br>Divide this number by 1000 and say your answer aloud. | Miss Wong, the teacher, has four cards. On each card is a number:<br><br>59 996   59 943   60 026   62 312<br><br>She gives one card to each pupil. The pupils look at their card and say a clue.<br>Anna says, 'My number is 60 000 to the nearest 10 thousand.'<br>Bashir says, 'My number has exactly 600 hundreds in it.'<br>Charis says, 'My number is 59900 to the nearest hundred.'<br>David says, 'My number is 60 000 to the nearest 10.'<br><br>Can you work out which card each pupil had? Explain your choices. |

This section reminds teachers to check pupils' understanding by asking questions such as 'Why', 'What happens if …', and checking that pupils can use the procedures or skills to solve a variety of problems.

This section contains examples of assessment questions, tasks and teaching activities that might support a teacher in assessing and evidencing progress of those pupils who have developed a sufficient grasp and depth of understanding so that learning is likely to be sustained over time.

This section contains examples of assessment questions, tasks and teaching activities that might support a teacher in assessing and evidencing progress of those pupils who have developed a stronger grasp and greater depth of understanding than that outlined in the first column.

www.mathshubs.org.uk
www.ncetm.org.uk
www.oxfordowl.co.uk

## Number and Place Value

### Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across 0
- solve number and practical problems that involve all of the above

### The Big Ideas

For whole numbers, the more digits a number has, the larger it must be: any 4-digit whole number is larger than any 3-digit whole number. But this is not true of decimal numbers: having more digits does not make a decimal number necessarily bigger. For example, 0·5 is larger than 0·35.

Ordering decimal numbers uses the same process as for whole numbers ie we look at the digits in matching places in the numbers, starting from the place with the highest value ie from the left. The number with the higher different digit is the higher number. For example, 256 is greater than 247 because 256 has 5 tens but 247 has only 4 tens. Similarly 1·0843 is smaller than 1·524 because 1·0843 has 0 tenths but 1·524 has 5 tenths.

### Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of  problems.

| Mastery | Mastery with Greater Depth |
|---|---|
| Think about the number 34 567 800.<br><br>Say this number aloud.<br>Round this number to the nearest million.<br><br>What does the digit '8' represent?<br>What does the digit '7' represent?<br><br>Divide this number by 100 and say your answer aloud.<br>Divide this number by 1000 and say your answer aloud. | Miss Wong, the teacher, has four cards. On each card is a number:<br><br>59 996 \| 59 943 \| 60 026 \| 62 312<br><br>She gives one card to each pupil. The pupils look at their card and say a clue.<br>Anna says, 'My number is 60 000 to the nearest 10 thousand.'<br>Bashir says, 'My number has exactly 600 hundreds in it.'<br>Charis says, 'My number is 59900 to the nearest hundred.'<br>David says, 'My number is 60 000 to the nearest 10.'<br><br>Can you work out which card each pupil had? Explain your choices. |

| Mastery | Mastery with Greater Depth |
|---|---|
| Put these numbers in order, from smallest to largest.<br>■ 3·3, 3·03, 3·33, 3·303, 3·033<br>■ 5834, 61·8 multiplied by 100, one tenth of 45813<br>■ 0·034, 3·6 divided by 100, ten times 0·0033<br>■ –4·4, –4·44, –4·04, –4·404 | Eduardo says, 'The the population of Mexico City is 11 million (to the nearest million) and the population of New York is 11·2 million (to the nearest hundred thousand).'<br>He says, 'The population of New York must be bigger than the population of Mexico City because 11·2 million is bigger than 11 million.'<br><br>Do you agree with him? |
| Estimate the answer to 4243 + 1734 by rounding the numbers to:<br>■ the nearest 1000<br>■ the nearest 100<br>■ the nearest 50<br>■ the nearest 10. | Three pupils are asked to estimate the answer to the sum 4243 + 1734.<br>Andrew says, 'To the nearest 100, the answer will be 5900.'<br>Bilal says, 'To the nearest 50, the answer will be 6000.'<br>Cheng says, 'To the nearest 10, the answer will be 5970.'<br><br>Do you agree with Andrew, Bilal or Cheng?<br>Can you explain their reasoning? |
| The population of Shanghai is 21 million, to the nearest million. Each person weighs on average 70 kg.<br><br>Estimate the total weight of all the people in Shanghai.<br><br>Do you think your answer is more or less than the actual answer you'd get if you weighed everyone in Shanghai accurately? | The total population of Shanghai is 21 million, to the nearest million.<br><br>If at lunchtime everyone in Shanghai eats a bowl of rice, how many grains of rice do you estimate are eaten each lunchtime? |

| Mastery | Mastery with Greater Depth |
|---|---|
| A scientist measures the depth of some objects below the surface of the sea. She records her measurements using negative numbers.<br><br>**Object** / **Depth**<br>Coral reef / –2 m<br>Shipwreck / –11 m<br>Pirate treasure / four times as deep as the coral reef<br>Sleeping shark / 3 metres above the shipwreck<br><br>Which object is deepest? Explain your choice.<br><br>Is the sleeping shark deeper than the pirate treasure? Explain your reasoning.<br><br>A seagull is hovering 1 m above the surface of the sea. How far apart are the seagull and the coral reef? | A scientist measured the temperature each day for one week at 06:00.<br><br>On Sunday the temperature was 1·6°C.<br>On Monday the temperature had fallen by 3°C.<br>On Tuesday the temperature had fallen by 2·1°C.<br>On Wednesday the temperature had risen by 1·6°C.<br>On Thursday the temperature had risen by 4·2°C.<br>On Friday the temperature had fallen by 0·9°C.<br>On Saturday the temperature had risen by 0·2°C.<br><br>What was the temperature on Saturday? |

| Object | Depth |
|---|---|
| Coral reef | –2 m |
| Shipwreck | –11 m |
| Pirate treasure | four times as deep as the coral reef |
| Sleeping shark | 3 metres above the shipwreck |

# Addition and Subtraction

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

**The Big Ideas**

Deciding which calculation method to use is supported by being able to take apart and combine numbers in many ways. For example, calculating 8·78 + 5·26 might involve calculating 8·75 + 5·25 and then adjusting the answer.

The associative rule helps when adding three or more numbers: 367 + 275 + 525 is probably best thought of as 367 + (275 + 525) rather than (367 + 275) + 525.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

| Mastery | Mastery with Greater Depth |
|---|---|
| Calculate 36·2 + 19·8<br>■ with a formal written column method<br>■ with a mental method, explaining your reasoning. | Jasmine and Kamal have been asked to work out 5748 + 893 and 5748 – 893.<br><br>Jasmine says, '893 is 7 less than 900, and 900 is 100 less than 1000, so I can work out the addition by adding on 1000 and then taking away 100 and then taking away 7.'<br><br>What answer does Jasmine get, and is she correct?<br><br>Kamal says, '893 is 7 less than 900, and 900 is 100 less than 1000, so I can work out the subtraction by taking away 1000 and then taking away 100 and then taking away 7.'<br><br>What answer does Kamal get, and is he correct?<br><br>If you disagree with either Jasmine or Kamal, can you correct their reasoning? |

| Mastery | Mastery with Greater Depth |
|---|---|
| Choose digits to go in the empty boxes to make these number sentences true.<br>$14\,781 - 6\square 53 = 8528$<br>$23{\cdot}12 + 22{\cdot}\square = 45{\cdot}23$ | Can you use five of the digits 1 to 9 to make this number sentence true?<br>$\square\square{\cdot}\square + \square{\cdot}\square = 31{\cdot}7$<br><br>Can you find other sets of five of the digits 1 to 9 that make the sentence true? |
| Two numbers have a difference of 2·38. The smaller number is 3·12.<br>What is the bigger number?<br><br>Two numbers have a difference of 2·3. They are both less than 10.<br>What could the numbers be? | Two numbers have a difference of 2·38. What could the numbers be if:<br>■ the two numbers add up to 6?<br>■ one of the numbers is three times as big as the other number?<br><br>Two numbers have a difference of 2·3. To the nearest 10, they are both 10.<br>What could the numbers be? |
| Compare $31 + 9 \times 7$ and $(31 + 9) \times 7$<br>What's the same? What's different?<br><br>Choose operations to go in the empty boxes to make these number sentences true.<br>$6 \square 3 \square 7 = 16$<br>$6 \square 3 \square 7 = 27$<br>$6 \square 3 \square 7 = 9$<br><br>Put brackets in these number sentences so that they are true.<br>$12 - 2 \times 5 = 50$<br>$12 - 8 - 5 = 9$<br>$10 \times 8 - 3 \times 5 = 250$ | Write different number sentences using the digits 2, 3, 5 and 8 before the equals sign, using:<br>■ one operation<br>■ two operations but no brackets<br>■ two operations and brackets.<br><br>Can you write a number sentence using the digits 2, 3, 5 and 8 before the equals sign, which has the same answer as another number sentence using the digits 2, 3, 5 and 8 but which is a different sentence? |

| Mastery | Mastery with Greater Depth |
|---|---|
| A shop sells magazines and comics. Freya buys a magazine and a comic. She pays £2·50. Evie buys a magazine and two comics. She pays £3·90.<br><br>How much does a comic cost? How much does a magazine cost? | A shop sells magazines and comics. Last week Arthur bought a magazine and a comic. He can't remember exactly what he paid, but he thinks he paid £1·76. Yesterday he bought a magazine and four comics. He paid £4·30.<br><br>Do you think he is remembering correctly when he says that he paid £1·76 last week? |
| A shop sells boxes of chocolates. One box costs £3·99. A second box costs £2·60. A third box costs £6·45.<br><br>What is the difference in price between the most and least expensive boxes?<br><br>The shop also sells packets of sweets. One packet costs £1·39. Ramesh has a £10 note and he wants to buy the chocolates costing £2·60.<br><br>How many packets of sweets can he also buy? | A shop sells boxes of chocolates costing £2·60. The shop also sells packets of sweets. One packet costs £1·39. Ramesh has a £10 note and he wants to buy one box of chocolates.<br>Sara says that Ramesh can work out how many packets of sweets he can buy using the number sentence $10 - 2{\cdot}60 \div 1{\cdot}39$.<br><br>Do you agree or disagree with Sara?<br>If you disagree, what number sentence do you think Ramesh should use?<br><br>Explain your reasoning. |
| *x* and *y* represent whole numbers.<br>Their sum is 1000.<br>*x* is 250 more than *y*.<br><br>What are the values of *x* and *y*? | *x* and *y* represent whole numbers. Their sum is 1000.<br><br>Can the difference between *x* and *y* be:<br>▪ 100?<br>▪ any whole number?<br>▪ greater than *x*? |

# Multiplication and Division

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication
- divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving addition, subtraction, multiplication and division
- multiply 1-digit numbers with up to two decimal places by whole numbers *(taken from Fractions including Decimals and Percentages)*

**The Big Ideas**

Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation.

Standard written multiplication method involves a number of partial products. For example, 36 × 24 is made up of four partial products 30 × 20, 30 × 4, 6 × 20, 6 × 4.

There are connections between factors, multiples and prime numbers and between fractions, division and ratios.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

## Mastery

Find numbers to complete these number sentences.

| | | |
|---|---|---|
| 736 ÷ 23 = □ | □ × 100 = 2400 | □ × 100 = 10 × □ |
| 7360 ÷ 230 = □ | 25 × □ = 200 | 25 × □ = 4 × □ |
| 230 × 24 = □ | 23 × □ = 161 | 23 × □ = 161 × □ |
| 240 × 23 = □ | 24 × □ = 168 | 24 × □ = 168 × □ |
| 1668 ÷ 8 = □ | 161 ÷ □ = 23 | 161 ÷ □ = 23 × □ |
| 208·5 × 8 = □ | □ ÷ 25 = 9 | □ ÷ 25 = 9 × □ |

## Mastery with Greater Depth

Fill in the missing numbers to make these number sentences true.

□ × □ = 864

□ × □ × □ = 864

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>It is correct that 273 × 32 = 8736. Use this fact to work out:<br>■ 27·3 × 3·2<br>■ 2·73 × 32 000<br>■ 873·6 ÷ 0·32<br>■ 87·36 ÷ 27·3<br>■ 8736 ÷ 16<br>■ 4368 ÷ 1·6</td><td>Which calculation is the odd one out?<br>■ 753 × 1·8<br>■ (75·3 × 3) × 6<br>■ 753 + 753 ÷ 5 × 4<br>■ 7·53 × 1800<br>■ 753 × 2 – 753 × 0·2<br>■ 750 × 1·8 + 3 × 1·8<br>Explain your reasoning.</td></tr>
<tr><td>Work out:<br>■ 8·4 × 3 + 8·4 × 7<br>■ 6·7 × 5 – 0·67 × 50<br>■ 93 × 0·2 + 0·8 × 93<br>■ 7·2 × 4 + 3·6 × 8</td><td>In each pair of calculations, which one would you prefer to work out?<br>■ (a) 35 × 0·3 + 35 × 0·7 or (b) 3·5 × 0·3 + 35 × 7<br>■ (c) 6·4 × 1·27 – 64 × 0·1 or (d) 6·4 × 1·27 – 64 × 0·027<br>■ (e) 52·4 ÷ 0·7 + 524 ÷ 7 or (f) 52·4 ÷ 0·7 – 524 ÷ 7<br>■ (g) 31·2 ÷ 3 – 2·4 ÷ 6 or (h) 31·2 ÷ 3 – 1·2 ÷ 0·3<br>Explain your choices.</td></tr>
<tr><td>All the pupils in a school were asked to choose between an adventure park and the seaside for a school trip.<br>They voted, and the result was a ratio of 5:3 in favour of the adventure park.<br>125 children voted in favour of going to the adventure park.<br><br>How many children voted in favour of going to the seaside?</td><td>All the pupils in a school were asked to choose between an art gallery and a science museum for a school trip.<br>The result was a ratio of 12:7 in favour of the science museum.<br>Five pupils were off school and didn't vote.<br>Every pupil went on the trip to the science museum the following week.<br>After the trip there is a news headline on the school website that says 'All 700 pupils in the school went to the science museum.'<br><br>Do you think that this news headline is correct? Explain your reasoning.</td></tr>
</table>

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>A box of labels costs £24.<br>There are 100 sheets in the box.<br>There are 10 labels on each sheet.<br><br>Calculate the cost of one label, in pence.</td><td>A box of labels costs £63.<br>There are 140 sheets in the box.<br>There are 15 labels on each sheet.<br>Sara, Ramesh and Trevor want to calculate the cost of one label, in pence.<br>Ramesh uses the number sentence $(6300 \div 140) \times 15$.<br>Sara uses the number sentence $63 \div 1{\cdot}4 \div 15$.<br>Trevor uses the number sentence $(15 \times 140) \div 6300$.<br><br>Who is using the right number sentence? Explain your choice.</td></tr>
<tr><td>Miriam and Alan each buy 12 tins of tomatoes.<br>Miriam buys 3 packs each containing 4 tins. A pack of 4 costs £1·40.<br>Alan buys 2 packs each containing 6 cans. A pack of 6 costs £1·90.<br><br>Who gets the most change from a £5 note?</td><td>Miriam buys 19 tins of soup. All the tins cost the same price.<br>She goes to the shop with just one note, and comes home with the tins and the change in coins. On the way home she drops the change. She looks carefully and she thinks she picks it all up. When she gets home she gives £2·23 change to her mother.<br><br>Do you think that Miriam picked up all the change that she dropped?<br><br>Explain your reasoning.</td></tr>
</table>

## Fractions and Decimals

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- use factors to simplify fractions; use common multiples to express fractions in the same denominator
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
- divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]
- multiply 1-digit numbers with up to two decimal places by whole numbers
- and use equivalences between simple fractions, decimals and percentages, including in different contexts

**The Big Ideas**

Fractions express a relationship between a whole and equal parts of a whole. Pupils should recognise this and speak in full sentences when answering a question involving fractions. For example, in response to the question 'What fraction of the journey has Tom travelled?' the pupil might respond, 'Tom has travelled two thirds of the whole journey.'

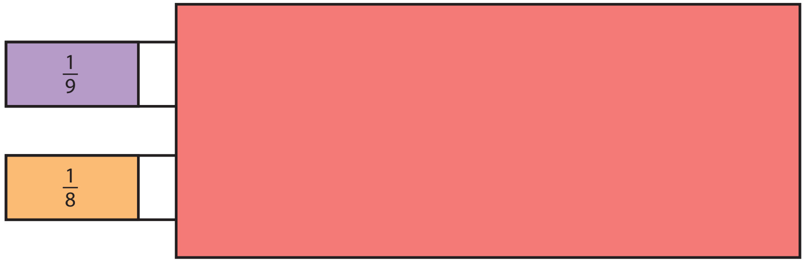
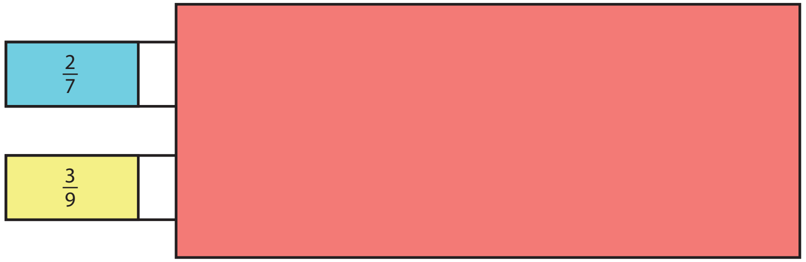
Equivalent fractions are connected to the idea of ratio: keeping the numerator and denominator of a fraction in the same proportion creates an equivalent fraction.

Putting fractions in place on the number lines helps understand fractions as numbers in their own right.

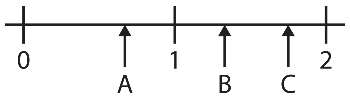
**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

| Mastery | Mastery with Greater Depth |
|---|---|
| Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.<br><br>Explain your reasoning. | Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer<br><br>Explain your reasoning. |
| In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true:<br>$\frac{1}{\square} = \frac{3}{\square}$<br>$\frac{\square}{3} = \frac{\square}{12}$<br>$\frac{\square}{\square} = \frac{\square}{\square}$<br>$\square \div \square = \square \cdot \square$<br>$\frac{30}{\square} = \frac{45}{\square}$ | Which is the odd one out?<br>$\frac{2}{5}$, 0·4, $\frac{4}{10}$, $\frac{3}{6}$, $\frac{6}{15}$<br>Explain your choice.<br><br>Put the following numbers into groups:<br>$\frac{3}{4}$, $\frac{3}{2}$, 0·5, 1·25, $\frac{3}{8}$, 0·125.<br>Explain your choices. |

| Mastery | Mastery with Greater Depth |
|---|---|
| Put the following numbers on a number line:<br>$\frac{3}{4}$, $\frac{3}{2}$, 0·5, 1·25, 3 ÷ 8, 0·125 | Suggest a fraction that could be at point A, a decimal that could be at point B and an improper fraction that could be at point C on this number line.<br> |
| On Monday I ran $1\frac{2}{3}$ km and on Tuesday I ran $2\frac{2}{5}$ km.<br>How far did I run altogether on these two days?<br><br>On Wednesday I ran $1\frac{2}{3}$ km and my sister ran $2\frac{2}{5}$ km.<br>How much further did my sister run than I did? | Altogether on Monday and Tuesday I ran $3\frac{1}{2}$ km. On neither day did I run a whole number of km.<br>Suggest how far I ran on Monday and how far on Tuesday.<br><br>On Wednesday I ran some km and my sister ran $1\frac{1}{6}$ km further than I did. Altogether we ran $4\frac{1}{2}$ km.<br><br>How far did I run on Wednesday? |

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>Sam added two fractions together and got $\frac{7}{8}$ as the answer.<br>Write down two fractions that Sam could have added.<br><br>Tom wrote down two fractions. He subtracted the smaller fraction from the larger and got $\frac{1}{5}$ as the answer.<br>Write down two fractions that Tom could have subtracted.<br><br>Tom and Sam shared equally one third of a chocolate bar.<br>What fraction of the chocolate bar did each child get?</td><td>Roland cuts a sandwich into two pieces. First, Roland gives one piece to Ayat and the other piece to Claire. Then Claire gives Ayat half of her piece. Now Ayat has $\frac{7}{8}$ of the original sandwich.<br><br>Did Roland cut the sandwich into two equal pieces? If not, how did he cut the sandwich?</td></tr>
<tr><td>Last month Kira saved $\frac{3}{5}$ of her £10 pocket money. She also saved 15% of her £20 birthday money.<br><br>How much did she save altogether?</td><td>Jakob says to Peter, ‘Last month I saved 0·5 of my pocket money and this month I saved $\frac{1}{3}$ of my pocket money, so altogether I’ve saved 40% of my pocket money’.<br><br>Do you think Peter should agree with Jakob?<br><br>Explain your decision.</td></tr>
<tr><td>What’s the same, and what’s different about these number statements?<br><br>Double one third of 15<br>One third of 30<br>$2 \times 5$<br>$15 \times 2 \div 3$<br>$15 \div 3 \times 2$<br>$15 \times \frac{2}{3}$</td><td>Amira says, ‘To work out a fraction of a number, you multiply the number by the numerator of the fraction and then divide the answer by the denominator of the fraction.’<br><br>Do you think that this is always, sometimes or never true?<br><br>Explain your reasoning.</td></tr>
</table>

| Mastery | Mastery with Greater Depth |
|---|---|
| In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true.<br>$\frac{1}{\square} \times \frac{3}{\square} = \frac{\square}{\square}$<br>$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{8}{15}$<br>$\frac{2}{\square} \times \frac{5}{\square} < \frac{10}{\square}$<br>$\frac{\square}{\square} \div 3 = \frac{1}{\square}$<br>$\frac{\square}{\square} \div 3 > \frac{1}{4}$ | True or false?<br>- The sum of two fractions is always greater than their product.<br>- If I divide a fraction by a whole number, the quotient is always smaller than the dividend.<br><br>Explain your reasoning. |
| Curtis used $\frac{1}{3}$ of a can of paint to cover 3·5 square metres of wall.<br><br>How much wall will one whole can of paint cover? | Puja shares 6 apples between some friends. Each friend gets 0·75 of an apple.<br><br>How many friends does she share the apples with? |

# Ratio and Proportion

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

**The Big Idea**

It is important to distinguish between situations with an additive change or a multiplicative change (which involves ratio). For example, if four children have six sandwiches to share and two more children join them, although two more children have been added, the number of sandwiches then needed for everyone to still get the same amount is calculated multiplicatively.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of  problems.

## Mastery

You can buy 3 pots of banana yoghurt for £2·40.

How much will it cost to buy 12 pots of banana yoghurt?

A child's bus ticket costs £3·70 and an adult bus ticket costs twice as much.

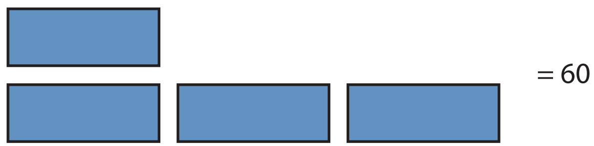
How much does an adult bus ticket cost?

To make a sponge cake, I need six times as much flour as I do when I'm making a fairy cake.

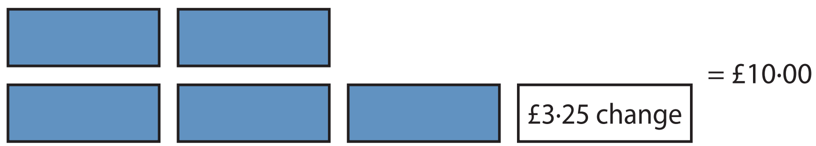
If a sponge cake needs 270 g of flour, how much does a fairy cake need?

## Mastery with Greater Depth

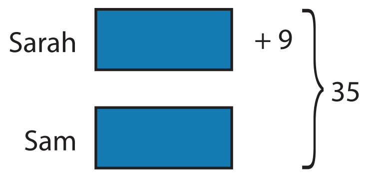
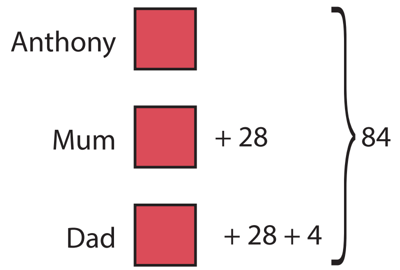
Make up a word puzzle that you could solve with this diagram:



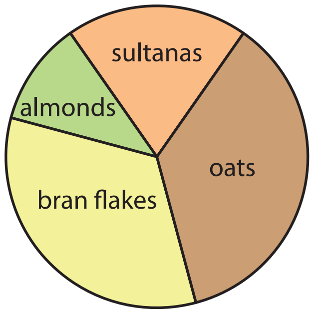
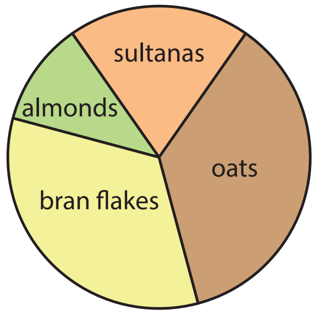

Make up a word puzzle that you could solve with this diagram:



<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>
Sam has 9 fewer sweets than Sarah. They have 35 sweets altogether.<br><br>
How many sweets does Sam have?<br><br>
*Bar modelling can be a useful strategy for solving these type of problems as illustrated below.*<br><br>
*For further information visit www.ncetm.org.uk/resources/44565*<br><br>

<br>
35 – 9 = 26<br>
26 ÷ 2 = 13<br>
Sam has 13<br>
Sarah has 22
</td><td>
Mum is 28 years older than Anthony. Mum is 4 years younger than Dad. The total age of the three of them is 84 years.<br><br>
What is Mum’s age?<br><br>
*Bar modelling can be a useful strategy for solving these type of problems as illustrated below.*<br><br>
*For further information visit www.ncetm.org.uk/resources/44565*<br><br>

<br>
84 – (28 + 28 + 4) = 24<br>
24 ÷ 3 = 8<br>
Mum is 8 + 28<br>
Mum is 36 years old
</td></tr>
<tr><td>If I share equally a 3 m ribbon between 5 people, how long will each person’s ribbon be?</td><td>I share equally a length of ribbon between 8 people, and each person gets 0·25m of ribbon.<br>Can you work out how long the original piece of ribbon was?</td></tr>
<tr><td>In Year 1 there are 50 pupils, of whom 16 are boys.<br><br>What percentage of the pupils are girls?</td><td>In a class of children 25% are boys and the rest are girls. There are 18 girls.<br><br>How many children are in the class?</td></tr>
</table>

| Mastery | Mastery with Greater Depth |
|---|---|
| Sam and Tom share 45 marbles in the ratio 2:3.<br><br>How many more marbles does Tom have than Sam? | Harry and Jim share some marbles in the ratio 3:5.<br>Harry gets 24 more marbles than Jim does.<br><br>How many marbles did they share in total between them? |
| To make a tomato pizza topping for a normal pizza, Jake uses 300 g of tomatoes, 120 g of onions and 75 g of mushrooms.<br>Jake wants enough sauce for a giant pizza, so he uses 900 g of tomatoes.<br><br>What mass of onions will be used?<br>How many 120 g boxes of mushrooms will he have to buy? | Jake has now made his giant pizza. He says, 'I made three times as much sauce to cover the giant pizza as I do to cover a normal pizza, so the giant pizza is three times as big as the normal pizza.'<br><br>Do you agree with Jake? |
| The pie chart shows the ingredients needed to make a breakfast cereal.<br><br>Estimate the percentage of the mixture that is sultanas.<br><br><br> | The pie chart shows the ingredients needed to make a breakfast cereal.<br>120 grams of almonds are used.<br><br>Estimate the quantity of each of the other ingredients.<br><br><br> |

## Algebra

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

**The Big Ideas**

A linear sequence of numbers is where the difference between the values of neighbouring terms is constant. The relationship can be generated in two ways: the sequence-generating rule can be **recursive**, i.e. one number in the sequence is generated from the preceding number (e.g. by adding 3 to the preceding number), or **ordinal**, i.e. the position of the number in the sequence generates the number (e.g. by multiplying the position by 3, and then subtracting 2).

Sometimes sequence generating rules that seem different can generate the same sequence: the ordinal rule 'one more than each of the even numbers, starting with 2' generates the same sequence as the recursive rule 'start at 1 and add on 2, then another 2, then another 2, and so on'.

Sequences can arise from naturally occurring patterns in mathematics and it is exciting for pupils to discover and generalise these. For example adding successive odd numbers will generate a sequence of square numbers.

Letters or symbols are used to represent unknown numbers in a symbol sentence (i.e. an equation) or instruction. Usually, but not necessarily, in any one symbol sentence (equation) or instruction, different letters or different symbols represent different unknown numbers.

A value is said to **solve** a symbol sentence (or an equation) if substituting the value into the sentence (equation) **satisfies** it, i.e. results in a **true statement**. For example, we can say that 4 **solves** the symbol sentence (equation) $9 - \triangle = \triangle + 1$ (or $9 - x = x + 1$) because it is a **true statement** that $9 - 4 = 4 + 1$. We say that 4 **satisfies** the symbol sentence (equation) $9 - \triangle = \triangle + 1$ (or $9 - x = x + 1$).

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

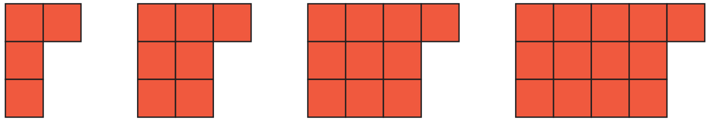
| Mastery | Mastery with Greater Depth |
| --- | --- |
| Ramesh is exploring two sequence-generating rules.<br>Rule A is: 'Start at 2, and then add on 5, and another 5, and another 5, and so on.'<br>Rule B is: 'Write out the numbers that are in the five times table, and then subtract 2 from each number.'<br><br>What's the same and what's different about the sequences generated by these two rules? | Ramesh is exploring three sequence-generating rules.<br>Rule A is: 'Start at 30, and then add on 7, and another 7, and another 7, and so on.'<br>Rule B is: 'Write out the numbers that are in the seven times table, and then add 2 to each number.'<br>Rule C is: 'Start at 51, and then add on 4, and another 4, and another 4, and so on.'<br><br>What's the same and what's different about the sequences generated by these three rules?<br><br>Explain why any common patterns occur. |
| Roshni and Darren are using sequence-generating rules.<br>Roshni's rule is: 'Start at 4, and then add on 5, and another 5, and another 5, and so on.'<br>Darren's rule is: 'Write out the numbers that are multiples of 5, starting with 5, and then subtract 1 from each number.'<br>Roshni and Darren notice that the first few numbers in the sequences generated by each of their rules are the same. They think that all the numbers in the sequences generated by each of their rules will be the same.<br><br>Do you agree? Explain your decision. | Roshni and Darren are using sequence-generating rules.<br>Roshni's rule is: 'Start at 5, and then add on 9, and another 9, and another 9, and so on.'<br>Darren's rule is: 'Write out the numbers that are multiples of 3, starting with 3, and then subtract 1 from each number.'<br><br>What might Roshni and Darren notice about the numbers in the sequences generated by each of these rules?<br><br>Explain your reasoning. |
| On New Year's Eve, Polly has £3·50 in her money box. On 1 January she puts 30p into her money box. On 2 January she puts another 30p into her money box. She continues putting in 30p every day.<br><br>How much money is in the box on 10 January?<br>How much money is in the box on 10 February?<br>Write a sequence-generating rule for working out the amount of money in the money box on any day in January. | On New Year's Eve, Polly has £3·50 in her money box. On 1 January she puts 30p into her money box. On 2 January she puts another 30p into her money box. She continues putting in 30p every day.<br><br>On what date is there exactly £8 in Polly's money box?<br>On what date does Polly's money box first contain more than £15?<br><br>Write a sequence-generating rule for working out the amount of money in the money box on any day. |

## Mastery

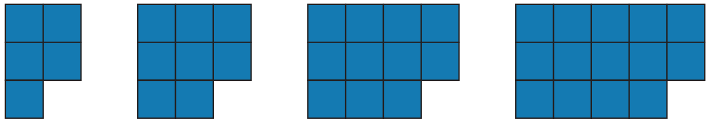
Ali has made three sequences of shapes by sticking coloured squares together.

The sequence of red shapes starts



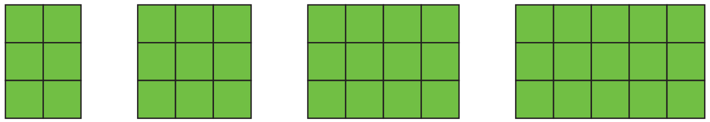
and so on.

The sequence of blue shapes starts



and so on.

The sequence of green shapes starts



and so on.

Ali says, 'If I put a red and a blue shape together, they will make a shape that is the same as one of the green shapes.'

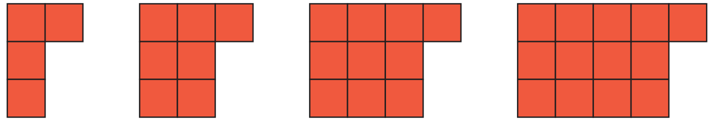
Do you agree with Ali?

Explain your reasoning.

## Mastery with Greater Depth

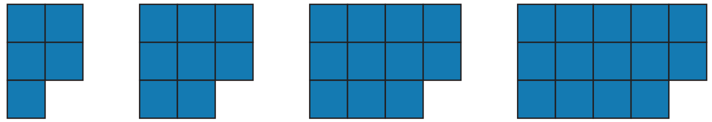
Ali has made three sequences of shapes by sticking coloured squares together.

The sequence of red shapes starts



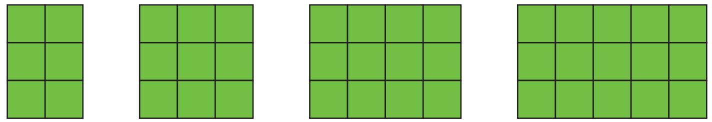
and so on.

The sequence of blue shapes starts



and so on.

The sequence of green shapes starts



and so on.

Ali says, 'If I put two shapes of the same colour together, they make a shape that is the same as one of the shapes in a different colour.'

Do you think that Ali's claim is always, sometimes or never true?

Explain your reasoning.

| Mastery | Mastery with Greater Depth |
|---|---|
| Which of the following statements do you agree with? Explain your decisions.<br>▪ The value 5 satisfies the symbol sentence $3 \times \square + 2 = 17$<br>▪ The value 7 satisfies the symbol sentence $3 + \square \times 2 = 10 + \square$<br>▪ The value 6 solves the equation $20 - x = 10$<br>▪ The value 5 solves the equation $20 \div x = x - 1$ | Which of the following statements do you agree with? Explain your decisions.<br>▪ There is a whole number that satisfies the symbol sentence $5 \times \square - 3 = 42$<br>▪ There is a whole number that satisfies the symbol sentence $5 + \square \times 3 = 42$<br>▪ There is a whole number that solves the equation $10 - x = 4x$<br>▪ There is a whole number that solves the equation $20 \div x = x$ |
| I am going to buy some 10p stamps and some 11p stamps.<br>I want to spend exactly 93p. Write this as a symbol sentence and find whole number values that satisfy your sentence.<br>Now tell me how many of each stamp I should buy.<br><br>I want to spend exactly £1·93. Write this as a symbol sentence and find whole number values that satisfy your sentence.<br>Now tell me how many of each stamp I should buy. | I am going to buy some 11p stamps and some 17p stamps.<br>I want to spend exactly 95p. Write this as a symbol sentence and find whole number values that satisfy your sentence.<br>Now tell me how many of each stamp I should buy.<br><br>I want to spend exactly £1·95. Write this as a symbol sentence and find whole number values that satisfy your sentence.<br>Now tell me how many of each stamp I should buy.<br><br>I want to spend exactly £1·59. Write this as a symbol number sentence.<br>Can you convince yourself that you can't find whole number values that satisfy your symbol sentence?<br><br>Explain your reasoning. |

## Measurement

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles

**The Big Ideas**

To read a scale, first work out how much each mark or division on the scale represents.

The unit of measure must be identified before measuring. Selecting a unit will depend on the size and nature of the item to be measured and the degree of accuracy required.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

| Mastery | Mastery with Greater Depth |
|---|---|
| Draw a clock face, then draw the hands showing that the time is 3 p.m.<br><br>Draw a second clock face, then draw the hands showing the time 12 000 seconds later. | Mehvish and Rima are looking at a clock face. They agree that at midday the hands of the clock lie on top of each other and so the angle between them is 0°. Rima thinks that at 3:15 p.m. the angle between the hands will be 90°. Mehvish thinks that the angle will be less than 90°.<br><br>Do you agree with Rima or Mehvish?<br><br>Explain your decision. |

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr>
<td>
A train left London at 09:46 and arrived in Edinburgh later that day.<br>
The clock in Edinburgh station showed this time:

How long did the train journey last?
</td>
<td>
Imagine we talked about time using decimals.<br>
Would 2·3 hours be:

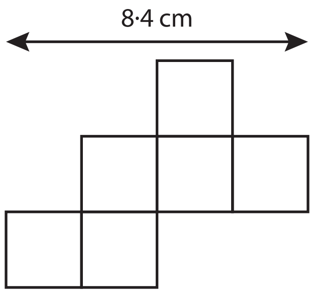
- 2 hours and 3 minutes
- 2 hours and 20 minutes
- 2 and a half hours, or
- 2 hours and 18 minutes?

Explain your decision.
</td>
</tr>
<tr>
<td>
Sarah is 0·2 m taller than Jack.<br>
Ella is 15 cm taller than Sarah.

Who is the tallest person?<br>
What is the difference in height between the tallest and the shortest person?
</td>
<td>
Sarah is 0·2 m taller than Jack.<br>
Ella is 15 cm taller than Sarah.<br>
Their combined height is 3·25 m.

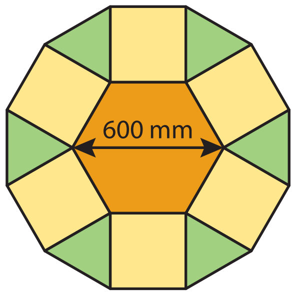
How tall is Ella?
</td>
</tr>
<tr>
<td>
Here is a tiled floor pattern.<br>
It is made from squares.

Work out the perimeter of the design.<br>
Give your answer in metres.



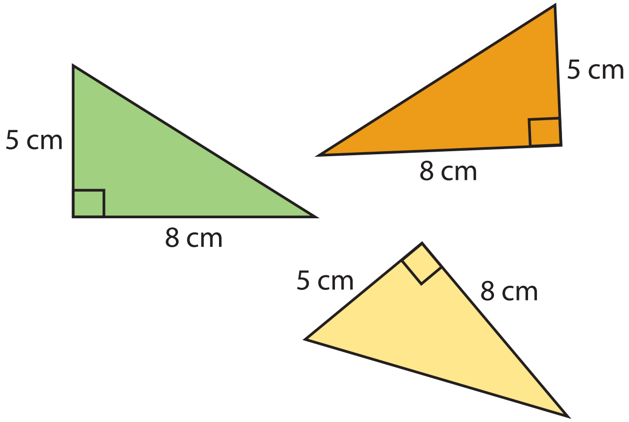
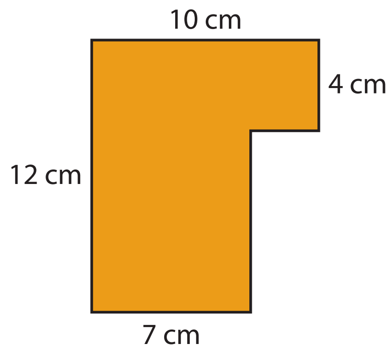
</td>
<td>
Here is a tiled floor pattern.<br>
It is made from equilateral triangles, squares and a regular hexagon.

Work out the perimeter of the design.<br>
Give your answer in metres.

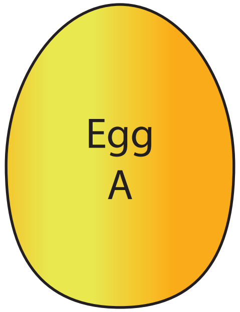
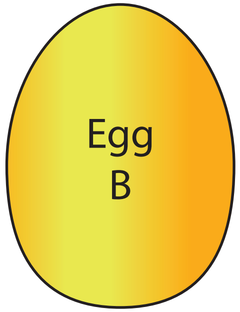


</td>
</tr>
</table>

| Mastery | Mastery with Greater Depth |
|---|---|
| Which of these right-angled triangles have an area of $20\,cm^2$?<br><br> | Sami worked out the area of the orange shape as $10 \times 4 + 8 \times 7 = 96\,cm^2$.<br>Razina worked out the area as $12 \times 7 + 3 \times 4 = 96\,cm^2$.<br>Lukas worked out the area as $10 \times 10 - 2 \times 2 = 96\,cm^2$.<br><br><br>Are you convinced by Sami, Razina or Lukas's reasoning?<br><br>Explain your answer. |
| Think about these rectangles:<br>■ a 4 cm by 6 cm rectangle<br>■ a 12 cm by 2 cm rectangle<br>■ a 3 cm by 8 cm rectangle.<br><br>What's the same? What's different? | Liping says, 'If you draw two rectangles and the second one has a greater perimeter than the first one, then the second one will also have a greater area.'<br><br>Do you agree or disagree with her?<br><br>Explain your reasoning. |
| The diameter of a golf ball is 4 cm. I want to make a box which will hold six golf balls.<br><br>What size could my box be?<br>Is there more than one answer? | Can you find two or more different cuboids each with a volume of $64\,cm^3$?<br>What's the same and what's different about your cuboids? |

| Mastery | Mastery with Greater Depth |
| --- | --- |
| 10 toy bricks have a total mass of 1 kg.<br>A cricket ball weighs $1\frac{1}{2}$ times as much as one brick.<br><br>What is the mass of a cricket ball, in grams? | In a story, Jack has to choose between two magic gold eggs to buy.<br>What would you advise him to do?<br><br>Mass when he buys it: 1·2 g<br>Mass doubles each day<br><br>Mass when he buys it: 125 g<br>Mass increases by 0·01 kg each day |

## Geometry

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles
- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes

**The Big Ideas**

Variance and invariance are important ideas in mathematics, particularly in geometry. A set of quadrilaterals for example may vary in many ways in terms of area, length of sides and the size of individual angles. However there are a set of invariant properties which remain common to all quadrilaterals, namely they have four sides and their internal angles sum to 360°. Some of these properties emerge from naturally occurring constraints, for example the sum of the internal angles will always sum to 360°, they can do nothing else! The questions 'What's the same?' and 'What's different?' can draw pupils' attention to variance and invariance.

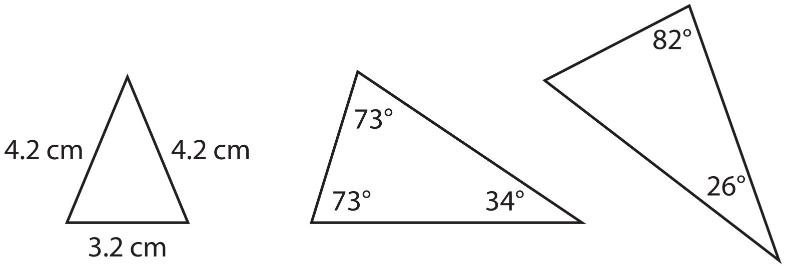
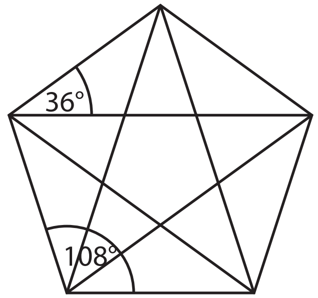
Shapes can be alike in essentially two different ways: congruent and similar. Congruent shapes are alike in all ways: they could occupy exactly the same space. Similar shapes share identical geometrical properties but can differ in size. All equilateral triangles are similar, but only identically sized ones are congruent. Not all isosceles triangles are similar.

Angle properties are a mix of necessary conditions and conventions. It is a necessary condition that angles on a straight line combine to a complete half turn. That we measure the half turn as 180° is conventional.

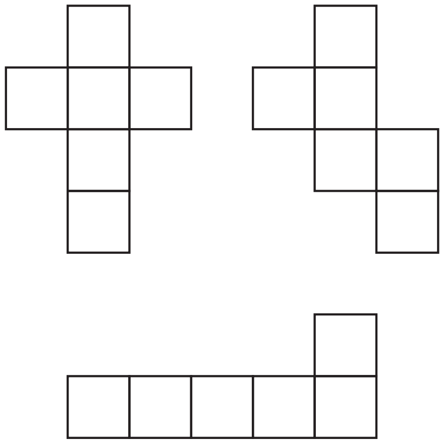
**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>Which of these triangles are isosceles?<br><br>Explain your decisions.<br><br></td><td>This is a regular pentagon.<br>Two angles (108° and 36°) are shown.<br><br>Which other angles can you work out?<br><br>Explain your reasoning.<br><br></td></tr>
<tr><td>Accurately draw two right-angled triangles with sides of different lengths.<br><br>Compare them and describe what's the same and what's different about them.</td><td>A triangle has been drawn carefully. You are told that the biggest angle is 20° larger than the second biggest angle and 40° larger than the smallest angle.<br><br>Work out how big each angle is.</td></tr>
<tr><td>A square has two vertices at (0,0) and (3,3).<br>Work out and explain the coordinates where the other two vertices could be.<br><br>A square has two vertices at (–3,0) and (3,0).<br>Work out and explain the coordinates where the other two vertices could be.</td><td>An isosceles triangle has two vertices at (–3,2) and (3,2).<br><br>Explore where the third vertex could be.</td></tr>
</table>

| Mastery | Mastery with Greater Depth |
|---|---|
| Captain Conjecture says, 'The diameter of a circle is twice the length of its radius.'<br><br>Do you agree?<br>Explain your answer.<br><br>Captain Conjecture says, 'All circles with a radius of 4 cm have circumferences that are the same length.'<br><br>Do you agree?<br>Explain your answer.<br> | Compare a circle and an oval.<br><br>What's the same and what's different? |
| Are these statements always, sometimes or never true?<br>▪ If a shape is reflected in an axis, it stays in the same quadrant.<br>▪ If a shape is translated to the right and up, it stays in the same quadrant.<br>▪ If a shape is translated to the left and down, it stays in the same quadrant.<br><br>Explain your decisions. | Joan says that if you reflect a shape (in an axis) and then reflect it again, the shape always ends up back where it first was as though you'd done nothing to it.<br><br>Do you agree with Joan?<br><br>Explain your decision. |
| Which of these could be the net of a cube?<br><br>Explain your choices.<br> | Pascal says that any net made with six squares can be folded to make a cube.<br><br>Do you agree with him?<br><br>Explain your reasoning. |

# Statistics

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- interpret and construct pie charts and line graphs and use these to solve problems
- calculate and interpret the mean as an average

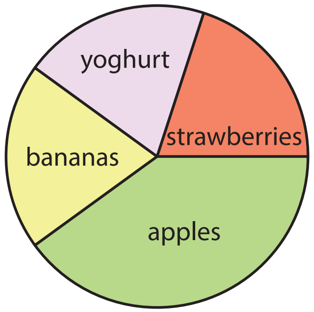
**The Big Ideas**

Pie charts visually display relative proportions, for example, that the proportion of pupils at School A liking reading is greater than the proportion at School B.

**Mastery Check**

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.

## Mastery



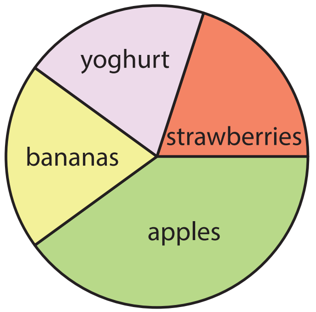

The pie chart represents the proportions of the four ingredients in a smoothie drink.

The sector representing the amount of strawberries takes up 22% of the pie chart.

The sector representing the amount of apple is twice as big as the sector representing the amount of strawberries.

The sectors representing the amount of yoghurt and the amount of banana are identical.

## Mastery with Greater Depth




The pie chart represents the proportions of the four ingredients in a smoothie drink.

The sector representing the amount of strawberries takes up 22% of the pie chart.

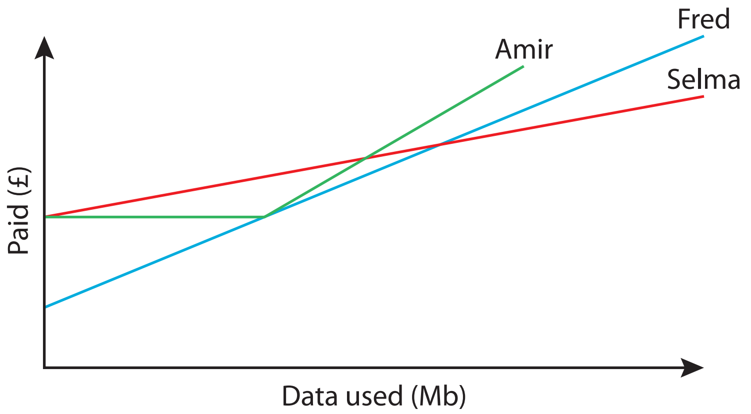
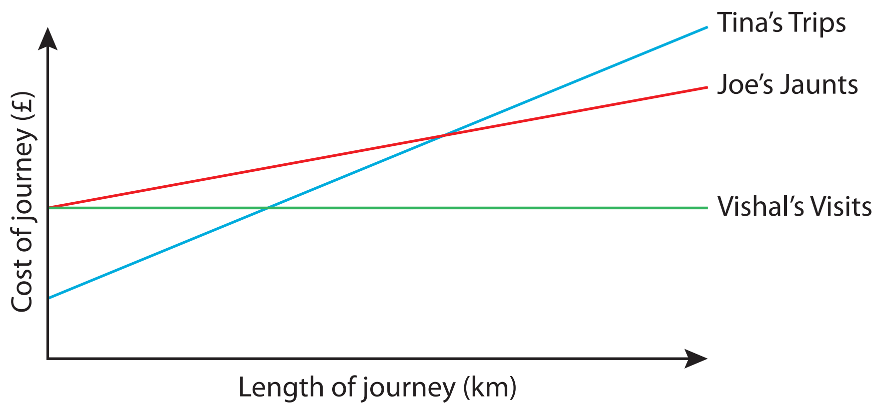
The sector representing the amount of apple is twice as big as the sector representing the amount of strawberries.

The sectors representing the amount of yoghurt and the amount of banana are identical.

<table>
<tr><th>Mastery</th><th>Mastery with Greater Depth</th></tr>
<tr><td>Calculate the percentage of bananas needed to make a smoothie drink.<br>What percentage of bananas would be needed to make two smoothie drinks?<br><br>Explain your reasoning.</td><td>Estimate the angle of the sector representing the amount of banana.<br><br>Explain your reasoning.</td></tr>
<tr><td>Ten pupils take part in some races on Sports Day, and the following times are recorded.<br>Time to run 100 m (seconds): 23, 21, 21, 20, 21, 22, 24, 23, 22, 20.<br>Time to run 100 m holding an egg and spoon (seconds): 45, 47, 49, 43, 44, 46, 78, 46, 44, 48.<br>Time to run 100 m in a three-legged race (seconds): 50, 83, 79, 48, 53, 52, 85, 81, 49, 84.<br><br>Calculate the mean average of the times recorded in each race.<br>For each race, do you think that the mean average of the times would give a useful summary of the ten individual times?<br><br>Explain your decision.</td><td>Three teams are taking part in the heats of a 4 × 100 m relay race competition on Sports Day. If the mean average time of the four runners in a team is less than 30 seconds, the team will be selected for the finals.<br>At the start of the last leg of the relay race, the times (in seconds) of each teams' first three runners are:<br>Team Peacock: 27, 29, 31<br>Team Farah: 45, 43, 37<br>Team Ennis: 29, 30, 25<br><br>Which of the teams have the best chance of being selected?<br><br>Explain your reasoning.</td></tr>
</table>

| Mastery | Mastery with Greater Depth |
|---|---|
| Three mobile phone companies each have different monthly pay-as-you-go contracts.<br>Phil's Phones: £5 fee every month and 2p for each Mb of data you use.<br>Manish's Mobiles: £7 fee every month and 1p for each Mb of data you use.<br>Harry's Handsets: £7 fee every month and 200Mb of free data, then 3p for each Mb of data after that.<br><br>Amir, Selma and Fred have mobile phones and they have recorded for one month how much data they have used (in Mb) and how much they have paid (in £). They have represented their data on this graph.<br><br><br><br><br>With which company do you think Amir has his contract?<br>With which company do you think Selma has her contract?<br>With which company do you think Fred has his contract?<br><br>Explain each of your choices. | Three taxi companies each work out the cost of a journey in different ways. I have taken lots of journeys with each of the companies, and have recorded each time how long the journey was (in km) and the cost of the journey (in £). I have represented these data on this graph.<br><br><br><br><br>What's the same and what's different about the ways in which the three companies work out the cost of a journey?<br>Which might you choose if you wanted to book a taxi to make a journey?<br>Explain your reasoning. |